TABLE I. Summary of masses and coupling constants for the mesons used in various proton-proton models. All the models include the one-pion contribution with $g_{\pi N}^2/4\pi=14.4$ and $m_\pi=135.1$ MeV, except BDR and Scotti-Wong who use $m_\pi=140$ MeV. For abbreviations, see text.

| Model | Meson | $g_s^2/4\pi$ | $g_v^2/4\pi$ | $g_T^2/4\pi$ | Mass (MeV) |
|---|---|---|---|---|---|
| BDR | Vector | | 30. | 0. | 760 |
| | Scalar | 13.8 | | | 560 |
| SUWY | Vector | | 1.2 | 13.2 | 540 |
| | Scalar | 2.4 | | | 405 |
| Scotti-Wong | $\rho$ | | 1.27 | 11.39 | 591 |
| | $\omega$ | | 2.77 | 0. | 780 |
| | Scalar | 1.525 | | | 437 |
| | $\eta$ | 12.12 | | | 550 |
| | $\varphi$ | | 2.26 | 0. | 1020 |
| ALV 2 | $\rho$ | | 0.84 | 11.6 | 750 |
| ALV 2+$\omega$ | $\rho$ | | 0.84 | 11.6 | 750 |
| | $\omega$ | | 13.6 | 0. | 780 |
| ALV 3+$\omega$ | $\rho$ | | 0.84 | 11.6 | 750 |
| | $\omega$ | | 16.9 | 0. | 780 |
| | Scalar | 5.16 | | | 400 |
| S1 | Scalar | 4.72 | | | 400 |
| S2 | Scalar | 3.22 | | | 400 |

data can be obtained from $\pi$, scalar, $\rho$ and $\omega$ exchange only if the "$2\pi$ basic" is neglected. If this is not done, then the only alternative to adding more mesons (adjustable parameters) is a more accurate treatment of the $N\bar{N}\rightarrow 2\pi$ $s$ and $p$ waves.

The calculations reported here were carried out at the Computation Center of The Pennsylvania State University.

## APPENDIX

The coupling constants listed in Table I were defined in terms of the interactions[2]:

$$\mathrm{Pi}=g_\pi\bar{\psi}i\gamma_5\psi\varphi,$$
$$\mathrm{Scalar}=g_s\bar{\psi}\psi\varphi,$$
$$\mathrm{Vector}=g_v\bar{\psi}i\gamma_\mu\psi\varphi_\mu+(g_T/4m)\bar{\psi}\sigma_{\mu\nu}\psi\varphi_{\mu\nu},$$

where:

$$\sigma_{\mu\nu}=1/2i(\gamma_\mu\gamma_\nu-\gamma_\nu\gamma_\mu),$$
$$\varphi_{\mu\nu}=\partial_\mu\varphi_\nu-\partial_\nu\varphi_\mu,$$
$$m=\text{mass of proton}.$$

Then $g_\pi^2/4\pi=14.4$.

PHYSICAL REVIEW VOLUME 135, NUMBER 4B 24 AUGUST 1964

# Prediction of a $\pi\eta$ Resonance

J. SCHECHTER AND S. OKUBO
*Department of Physics and Astronomy, University of Rochester, Rochester, New York*
(Received 9 April 1964)

Using the bootstrap approximation of Zachariasen and Zemach, we predict a relatively broad $1^{--}$ resonance at about 1 BeV in a two-channel ($\pi\eta,\pi\rho$) calculation. Such a resonance can only belong to the isocuplet representation of SU(3). Tentatively an identification can be made on this basis with the $B$ meson, and a recently observed peaking in the $\pi\rho$ channel at about 1250 MeV.

## 1. INTRODUCTION

THE pseudoscalar mesons—$\pi K\bar{K}\eta$—are now considered to belong to the octet representation of SU(3). A scattering state of two pseudoscalar mesons can be any of the 1, 8, 8′, 10, $\overline{10}$, or 27 dimension representations. For $p$-wave scattering, Bose statistics allow only the 8′, 10, and $\overline{10}$ states. It is observed that the well-known $p$-wave resonances—$\rho K^*\bar{K}^*\omega_8$—can be classified according to the 8′ representation. Naturally it is of interest to investigate whether the 10 and $\overline{10}$ $p$-wave states do not also resonate. In this connection Neville[1] has noticed that within the framework of a single-channel bootstrap calculation where all pseudoscalar and vector-meson masses are taken, respectively, equal, the tenfold resonant states should exist at the same mass as the observed octet state. We investigate this situation in somewhat more detail by looking at $\pi\eta$ scattering. The $\pi\eta$ state with odd parity cannot belong to 1, 8, 8′, or 27, but as we shall see must belong to the icosuplet state which consists of the 10 and its antiparticle state $\overline{10}$. Since this state is part of the multiplet, any general conclusion drawn about it should be valid for the entire multiplet. However, since in this note we do not look at all the decuplet channels, the calculation is to a large extent independent of SU(3).

We use the bootstrap technique to study the $l=1$, $J=1$, $I=1$, $G=-1$ amplitude. This method has the following history: Recently, Zachariasen[2] noticed that the appearance of the $\rho$ resonance in $\pi\pi$ scattering could be qualitatively explained if it were assumed that the dominant force in this scattering comes from the exchange of a $\rho$. By taking the additional channel $\pi\pi\rightarrow\pi\omega$

[1] D. E. Neville, Phys. Rev. **132**, 844 (1963).

[2] F. Zachariasen, Phys. Rev. Letters **1**, 112 (1961); **1**, 268 (E) (1961).

into account Zachariasen and Zemach[3] were able to obtain a fairly good semiquantitative understanding of the $\rho$ meson. Similarly, Capps[4] was able to explain the $K^*$ meson by using the bootstrap approach. He looked at the coupled $\pi K-\eta K$ system. The success of these calculations makes it appear reasonable that a similar approach should be valid for the scattering of any two pseudoscalar mesons.

First a one-channel calculation is made. This is equivalent to considering just the $\pi$ and $\eta$ particles. We should emphasize that the transition $\pi\eta\to K\bar{K}$ is forbidden in the odd-parity, $I=1$ state because of $G$-conjugation invariance. Hence, this one-channel calculation is essentially equivalent to the full SU(3) calculation of the decuplet resonant $p$ state where we neglect other channels involving the vector octet. A self-consistent theory can be constructed if a resonance, designated by $V$, exists and provides the main contribution to the force. We note that the exchange of a $\rho$, $\omega$, or $\phi$ is not allowed in $\pi\eta$ scattering. The equations are sufficient to determine the mass of $V$ and the $\pi\eta V$ coupling constant. The results are dependent on a subtraction point. A very broad resonance is found at about 1.1 BeV.

Next, a two-channel calculation is made, the possibility $\pi\eta\to\pi\rho$ being taken into account. A self-consistent solution with $V$ is found and in addition to the mass of $V$ the $\pi\eta V$ and $\pi\rho V$ coupling constants are obtained from the consistency equations. $V$ turns out to be a broad resonance (width$\simeq$125 MeV) at about 950 MeV. The results are less dependent on the choice of a subtraction point than in the one-channel case. The above solution is obtained by considering all forces to arise from exchange of a $V$. In the $\pi\rho\to\pi\rho$ scattering, however, the forces from $\pi$ and $\omega$ exchange are also expected to contribute. We estimate using conventional values for the $\rho\pi\pi$ and $\rho\pi\omega$ coupling constants that these forces do not greatly change the solution. In this connection, we have neglected the channel $\pi\eta\to K\bar{K}^*$. This may modify the position of the resonance.

## 2. THE SELF-CONSISTENCY EQUATIONS

The theory of the bootstrap equations is given in the paper of Zachariasen and Zemach.[3] We review this briefly from the standpoint of establishing our notation and emphasizing those points which are relevant to our case.

We wish to construct the low-energy $J=1$ scattering amplitudes for the following processes:

$$\pi\eta\to\pi\eta\,,$$
$$\pi\eta\leftrightarrow\pi\rho\,,$$
$$\pi\rho\to\pi\rho\,.$$

In order to keep the calculation within manageable bounds, the higher mass channels with the same quantum numbers—$K^*\bar{K}$, $K\bar{K}^*$, $\eta V$—are neglected. The $K\bar{K}$ and $\pi\omega$ channels have the wrong $G$ parity to communicate with $\pi\eta$. The $\pi\eta$, $\pi\rho$, $K^*\bar{K}$, and $\eta V$ threshold energies are 690, 900, 1380, and 1500 MeV, respectively, so that neglecting the last two does not seem unreasonable.

The amplitudes form a $2\times2$ matrix $T_{fi}$, where the $\pi\eta$ channel is designated channel 1 and the $\pi\rho$ channel is designated channel 2. $T_{fi}$ is required to satisfy in approximation the following four conditions:

(i) Elastic unitarity:

$$\operatorname{Im}T_{fi}^{-1}(x)=\frac{q_i\delta_{fi}\theta(x-x_i)}{2(4\pi)^2x^{1/2}}\,, \tag{1}$$

where $x=s/(m_\pi+m_\eta)^2$, $x_i=$threshold for channel $i$ ($x_1=1$, $x_2\simeq1.70$), $q_i=$center-of-mass momentum for channel $i$.

(ii) Analyticity: This will be satisfied in the sense that of all the singularities[5] of $T_{fi}$ in the complex $x$ plane, only those due to unitarity and the exchange of single particles will be considered. The $V$ resonance will be considered to produce a pole on an unphysical sheet. An analytic solution which also satisfies (i) can be obtained by using the matrix $N/D$ method. After defining $t_{fi}=q_fq_iT_{fi}$ in order to eliminate threshold kinematic singularities, we write $t_{fi}=\sum_K N_{fK}D_{Ki}^{-1}$. With the assumption above, the first iteration yields:

$$N_{ji}(x)=q_jq_iT_{ji}^B(x)\,, \tag{2a}$$

$$D_{ji}(x)=\delta_{ji}+\frac{x-x_0}{\pi}\int_{x_j}^{\infty}\frac{dx'}{2(4\pi)^2}\frac{q_i(x')}{(x')^{1/2}}\times\frac{T_{ji}^B(x')}{(x'-x_0)(x'-x-i\epsilon)}\,, \tag{2b}$$

where $x_0=$subtraction point and $T_{ji}^B(x)$ is the Born-approximation amplitude calculated from the one-particle-exchange diagram for the relevant scattering process.

The disadvantage of this method of solution, as has been noted by many authors, is that it doesn't guarantee $T_{ij}=T_{ji}$ required by time-reversal invariance.

(iii) Crossing symmetry: This will be satisfied very crudely by choosing $x_0$ appropriately. When $x=x_0$, $T_{ji}(x_0)=T_{ji}^B(x_0)$. Thus $x_0$ should be chosen in a region where the contribution of the one-particle-exchange diagram to the singularities in $T$ is dominant. Figure 1 shows the branch cuts resulting from the exchange of a $V$ with mass squared$=R$. The exchange of higher mass systems can be described by the same figure with larger $R$. These systems therefore are expected to make dominant contributions further to the left. It is clear that a believable solution should have the property that it not change much upon varying the subtraction point in the

[3] F. Zachariasen and C. Zemach, Phys. Rev. **128**, 849 (1962).
[4] R. H. Capps, Phys. Rev. **131**, 1307 (1963).
[5] J. Kennedy and T. D. Spearman, Phys. Rev. **126**, 1596 (1962).

region slightly to the left of the origin. We shall see that the solution obtained from a single-channel claculation is much more ambiguous than the one obtained in the two-channel case. This point has been emphasized by Diu, Gervais, and Rubenstein.[6]

(iv) The resonance $V$ must appear in all scattering processes; near $x=R$ $t_{ij}$ must have the form

$$t_{ij}=a_{ij}/[(x-R)+iR^{1/2}\Gamma], \quad (3)$$

where $\Gamma$ = full width of resonance and $a_{ij}$ may be calculated from the graphs of Fig. 2.

Only two of the four equations given in (3) are independent. These and the condition that the real part of the denominator of $t_{ij}$ as computed from Eq. (2) vanishes comprise the self-consistency equations. A solution of them determines $R$ and two coupling constants provided suitable other information is known. Explicitly then, we require

$$\mathrm{Re}\Delta(R)=\mathrm{Re}[\det D(R)]=0, \quad (4)$$

$$a_{11}=(N_{11}(R)\,\mathrm{Re}D_{22}(R)-N_{12}(R)\,\mathrm{Re}D_{21}(R))/\Delta_R', \quad (5a)$$

$$a_{22}=(N_{22}(R)\,\mathrm{Re}D_{11}(R)-N_{21}(R)\,\mathrm{Re}D_{12}(R))/\Delta_R', \quad (5b)$$

where

$$\Delta_R'=\left[\frac{d}{dx}\mathrm{Re}\Delta(x)\right]_{x=R}.$$

In writing Eqs. (5) the assumption $\mathrm{Im}D_{ij}\ll\mathrm{Re}D_{ij}$ is made. From Eqs. (2) and (3) we obtain an expresssion for the width of $V$:

$$\Gamma=(1/32\pi^2R)[(a_{11}/q_1)\theta(R-x_1)+(a_{22}/q_2)\theta(R-x_2)]. \quad (6)$$

The first term in Eq. (6) is the partial width for $V\to\pi\eta$ while the second term is the partial width for $V\to\pi\rho$. If $\Gamma$ turns out unreasonably large, the assumption that $t_{ij}$ has a resonance at $x=R$ is not a good one.

### 3. CALCULATION OF GRAPHS

In this section we give the $J=1$, $I=1$ contributions of the necessary resonance and one-particle-exchange graphs. The following equivalent interaction Lagrangian density is used:

$$L_I=2\gamma_{\rho\pi\pi}\boldsymbol{\rho}_\mu\times\boldsymbol{\pi}\cdot\partial_\mu\boldsymbol{\pi}+\frac{\gamma_{\rho\pi\omega}}{m_\pi}\epsilon_{\alpha\beta\gamma\delta}\partial_\alpha\boldsymbol{\pi}\cdot\partial_\beta\boldsymbol{\rho}_\gamma\omega_\delta$$
$$+2\gamma_{\pi\eta V}\mathbf{V}_\mu\cdot(\boldsymbol{\pi}\partial_\mu\eta-\partial_\mu\boldsymbol{\pi}\eta)$$
$$+\frac{\gamma_{\pi\rho V}}{m_\pi}\epsilon_{\alpha\beta\gamma\delta}\partial_\alpha\boldsymbol{\pi}\times\partial_\beta\boldsymbol{\rho}_\gamma\cdot\mathbf{V}_\delta. \quad (7)$$

The diagonal elements of $a$ in Eq. (3) are

$$a_{11}=G_\eta^2q_1^4(R),$$
$$a_{22}=G_\rho^2Rq_2^4(R),$$

where

$$G_\eta=8\left(\frac{\pi}{3}\right)^{1/2}\gamma_{\pi\eta V},$$

$$G_\rho=4\left(\frac{\pi}{3}\right)^{1/2}\gamma_{\pi\rho V}/M_\pi, \quad (8)$$
$$q_1^2(x)=[(x-1)/4x][x-(M_\eta-M_\pi)^2],$$
$$q_2^2(x)=(1/4x)[x-(M_\rho+M_\pi)^2][x-(M_\rho-M_\pi)^2],$$
$$M_i=m_i/(m_\pi+m_\eta).$$

The Born approximation for $\pi\eta\to\pi\eta$ is obtained from the graph with $V$ exchange:

$$F_{11}=\frac{N_{11}}{G_\eta^2}=\tfrac{3}{8}E\left[2-B\ln\frac{B+1}{B-1}\right],$$
$$B=(1/4q_1^2)[2(M_\eta^2+M_\pi^2) \quad (9)$$
$$-2R-x+(1/x)(M_\eta-M_\pi)^2],$$
$$E=\tfrac{1}{2}[R+2x-2(M_\eta^2+M_\pi^2)-(1/R)(M_\eta-M_\pi)^2].$$

$\rho$ exchange does not contribute to $\pi\eta$ scattering because of $G$-parity conservation, while $\omega$ and $\phi$ exchange do not contribute because of isotopic-spin conservation.

Similarly a single $V$ exchange contributes to the Born approximation for $\pi\eta\leftrightarrow\pi\rho$.

$$F_{12}=N_{12}/G_\eta G_\rho=N_{21}/G_\eta G_\rho$$
$$=-\tfrac{3}{8}q_1q_2x^{1/2}\left[2C+(1-C^2)\ln\frac{C+1}{C-1}\right],$$
$$x>(M_\rho+M_\pi)^2$$
$$=-\tfrac{3}{8}q_1|q_2|x^{1/2}[2|C|-2(1+|C|^2)\arctan(1/|C|)],$$
$$1<x<(M_\rho+M_\pi)^2,$$

$$C=\frac{1}{2q_1q_2}\{2[(q_1^2+M_\pi^2)(q_2^2+M_\rho^2)]^{1/2}$$
$$+R-(M_\pi^2+M_\rho^2)\}. \quad (10)$$

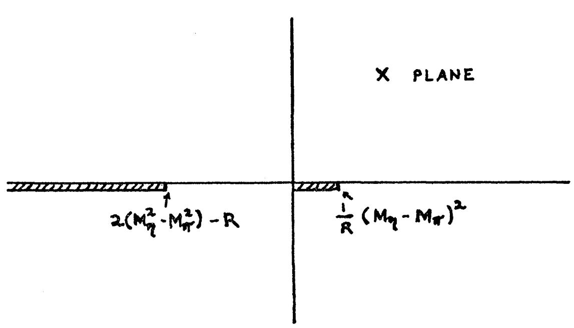
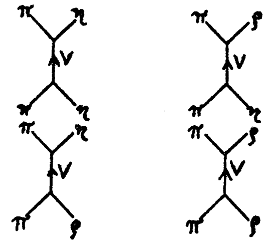



FIG. 1. Born-approximation branch cuts.

[6] B. Diu, J. L. Gervais, and H. R. Rubenstein, Nuovo Cimento **31**, 341 (1964).

FIG. 2. Resonance graphs.




The contribution from $V$ exchange to $\pi\rho$ scattering is

$$F_{22}{}^{V}=\frac{N_{22}{}^{V}}{G_\rho{}^2}=\frac{-3}{32}q_2{}^2\left[(A_0-GD)\ln\frac{D+1}{D-1}+2G-\tfrac{2}{3}q_2{}^2\right], \quad (11)$$

where

$$D=\frac{1}{4q_2{}^2}\left[x+2R-2(M_\rho{}^2+M_\pi{}^2)-\frac{1}{x}(M_\rho-M_\pi)^2\right],$$

$$G=A_1-A_2D-q_2{}^2D^2,$$

$$A_0=q_2{}^2+M_\pi{}^2,$$

$$A_1=2x-3q_2{}^2-2(M_\rho{}^2+M_\pi{}^2),$$

$$A_2=3q_2{}^2+2M_\rho{}^2+M_\pi{}^2.$$

With $N_{22}{}^V$, Eq. (2b) for $D_{22}$ diverges. Since the divergence is only logarithmic, the choice of a physically reasonable cutoff is not too critical. We use a cutoff of about 5 nucleon masses.

Taking[3] $\gamma_{\rho\pi\omega}{}^2/4\pi=0.15$, we find for the $\omega$-exchange force in $\pi\rho$ scattering

$$N_{22}{}^{\omega}=75q_2{}^2\left[(A_0-GD)\ln\frac{D+1}{D-1}+2G-\tfrac{2}{3}q_2{}^2\right], \quad (12)$$

where $D$, $G$, $A_0$, $A_1$, and $A_2$ are the same as in Eq. (11) except that $R$ is to be replaced by $M_\omega{}^2$.

With[3] $\gamma_{\rho\pi\pi}{}^2/4\pi=0.5$ the $\pi$-exchange Born term is

$$N_{22}{}^{\pi}=-158q_2{}^2\left[(1-D_\pi{}^2)\ln\frac{D_\pi+1}{D_\pi-1}+2D_\pi\right], \quad (13)$$

where

$$D_\pi=(1/4q_2{}^2)[2M_\rho{}^2-x+(1/x)(M_\rho-M_\pi)^2].$$

### 4. THE ONE-CHANNEL EQUATIONS

For orientation purposes it is useful to look at the single-channel case. Here the two self-consistency equations can be approximated by an equation which clearly shows the physical interpretation and the defects involved. From Eq. (2b) we may write

$$\mathrm{Re}D_{11}=1-[(x-x_0)/32\pi^3]H_{11}G_\eta{}^2,$$

where

$$H_{11}=P\int_1^\infty \frac{F_{11}(x')}{(x'-x_0)(x'-x-i\epsilon)}\frac{dx'}{q_1(x')(x')^{1/2}}.$$

The consistency equation (4) becomes in this case:

$$G_\eta{}^2=[-32\pi^3/(R-x_0)][1/H_{11}(R)]. \quad (14)$$

Using Eq. (8), consistency equation (5a) goes to:

$$G_\eta{}^2=\frac{F_{11}(R)}{q_1{}^4(R)}\frac{32\pi^3}{H_{11}(R)+(R-x_0)H_{11}'(R)}$$

$$\simeq\frac{F_{11}(R)}{q_1{}^4(R)}\frac{32\pi^3}{H_{11}(R)}, \quad (15)$$

where

$$H_{11}'(R)=[(d/dx)H_{11}(x)]_{x=R}.$$

Numerically we observe that $(R-x_0)H_{11}'(R)/H_{11}(R)<0.1$ in our case. Equations (14) and (15) have a simultaneous solution at that value of $R$ for which

$$F_{11}(R)/q_1{}^4(R)=-1/(R-x_0). \quad (16)$$

This is a simple relation and does not involve the evaluation of integrals. (However, this task cannot be escaped if $G_\eta{}^2$ is to be found.) Indeed in the energy region just above threshold the behavior of $F_{11}(R)$ is as $fq_1{}^4(R)$, where $f$ is a number measuring the strength and sign of the "Born force." With our conventions $F_{11}$ must be negative for a possible resonance and hence for an attractive force. The necessary magnitude of $F_{11}$ is made somewhat ambiguous by the fact that the value of the subtraction point $x_0$ is not uniquely fixed. The one-channel, single-exchange-diagram input bootstrap may therefore be best thought of as providing a rough guide to the question of the existence of a resonance. Equation (6) for the width is here

$$\Gamma=G_\eta{}^2q_1{}^3(R)/32\pi^2R. \quad (17)$$

Using $x_0=-0.5$, a solution is obtained at $R=2.5$ (1.1 BeV) and gives $\Gamma\simeq320$ MeV. Using $x_0=-1.0$ we obtain a solution at about 1.3 BeV with $\Gamma\simeq690$ MeV. The indication is that the force supplied by $V$ exchange is barely sufficient to sustain a rather broad resonance. Therefore it is necessary to investigate the effects of other forces.

### 5. THE TWO-CHANNEL EQUATIONS

Since no other single-particle exchange forces contribute to $\pi\eta$ scattering we expect the inclusion of the possibility $\pi\eta\leftrightarrow\pi\rho$ to take account of the next most important forces. The simplest way to do this is to treat the $\rho$ as a metastable particle and to use the two-channel formulation. Since the $\rho$ decays so quickly, there is some doubt about doing this, but in any case we hope that the $\pi\rho$ state so described will at least approximate the behavior of a $J=1$, $I=1$ $3\pi$ state.

First we calculated this two-channel case taking only $V$ exchange into account in the $\pi\rho\to\pi\rho$ process. The three consistency equations are Eqs. (4), (5a), and (5b). The $N_{ij}$ are obtained from Eqs. (9), (10), and (11), the necessary $a_{ij}$ are given in Eq. (8), and the $\mathrm{Re}D_{ij}$ are found by performing principal value integrations in Eq. (2b). We solved Eqs. (4), (5a), and (5b) simultaneously for $\gamma_{\pi\eta V}^2$ and $\gamma_{\pi\rho V}^2$ as defined in Eq. (7) and for $R$. The method used was to obtain for each value of $R$ three curves for $\gamma_{\pi\rho V}^2$ as a function of $\gamma_{\pi\eta V}^2$ and to look for a point of intersection. Since *a priori* there is no reason to expect a solution to exist, the fact that one which is relatively stable with respect to varying the subtraction point is obtained is encouraging. Table I

TABLE I. Results of 2-channel calculation for several subtraction points.

| $x_0$ | $-1.0$ | $-0.5$ | 0 |
|---|---|---|---|
| $\gamma^2_{\pi\eta V/4\pi}$ | 0.95 | 1.3 | 2.1 |
| $\gamma^2_{\pi\rho V/4\pi}$ | 0.72 | 0.61 | 0.37 |
| mass of $V$ in MeV | 930 | 940 | 970 |
| $\Gamma$ in MeV | 91 | 126 | 203 |
| ratio: $\pi\rho/\pi\eta$ | 0.37 | 0.31 | 0.20 |

summarizes the results for three reasonable subtraction points. In addition the width of $V$ and the ratio of $\pi\rho$ to $\pi\eta$ partial widths as calculated from Eq. (6) is listed for each point. Comparing with the results of the one-channel case we see that there is less sensitivity here to changes in $x_0$.

Next we estimate the effects of $\pi$ and $\omega$ exchange in the $\pi\rho\to\pi\rho$ process. The effect of $\phi$ exchange should be small compared to the effect of $\omega$ exchange because the decay $\phi\to\rho+\pi$ is relatively weak. Equation (13) gives the Born term for $\pi$-exchange scattering. Numerically,

$$D_\pi \simeq [1/4q_2^2(x)][2.4+(1.37/x)-x].$$

We note that for $x\simeq 2.5$, $D_\pi=1$ and for larger $x$ becomes smaller, eventually going negative. Thus $\ln(D_\pi+L)/(D_\pi-1)$ diverges at $x\simeq 2.5$ and is a complex quantity in a region just above 2.5. This is because the exchanged pion is physical rather than virtual. This may contribute to $\pi\rho\to 3\pi$ instead of $\pi\rho\to\pi\rho$. In any event the resonance we found was at $x\simeq 2.0$, somewhat below the dangerous region. Actually, in the region where the resonance appears, the contributions from $\omega$ and $\pi$ exchange tend to cancel, the $\omega$ exchange being repulsive while the $\pi$ exchange is attractive. There is thus some justification for neglecting the $\pi$ and $\omega$ exchanges, This justification will be better if the over-all solution is not too dependent on perturbations in the $\pi\rho$ channel. To test this we calculated the two-channel case taking $V$ *and* $\omega$ exchange into account and found very little change in the solution. We found for $x_0=-\frac{1}{2}$: $R=940$ MeV, $\gamma^2_{\pi\eta V/4\pi}=1.3$, $\gamma^2_{\pi\rho V/4\pi}=0.7$, $\Gamma\simeq 130$ MeV, and $\pi\rho/\pi\eta=0.35$. It then seems that relatively large changes in the $\pi\rho$ channel produce small changes in the calculated parameters of $V$.

We therefore consider that our calculation has made plausible the existence of a $1^{--}$ resonance at about 1 BeV in the $\pi\eta$, $\pi\rho$ scattering channels. It is in the spirit of previous bootstrap calculations to regard the results as semiquantitative rather than quantitative.

## 6. CONNECTION WITH SU(3) INVARIANCE

We assume that the state $V$ belongs to some irreducible representation of SU(3).[7] Since $V$ is composed of $\pi$ and $\eta$ in a relative $p$ state the SU(3) "wave function" describing $V$ must be antisymmetric on interchange of $\pi$ and $\eta$. Of the product representations which can be formed from two octets only $8'$, 10, and $\overline{10}$ are antisymmetric. It is seen by explicit calculation that the $\pi\eta$ state does not occur in $8'$. Thus $V$ must belong to 10, $\overline{10}$ or their linear combination. However, because $V$ has definite $G$ parity the antiparticle of, for example, $V^+$ must be $V^-$. On the other hand, the antiparticles of 10 lie in $\overline{10}$. Therefore $V$ cannot belong to either 10 or $\overline{10}$ but must correspond to the combination[8]

$$V=(1/\sqrt{2})(\psi-G\psi), \tag{18a}$$

where $\psi$ is the $I=1$, $Y=0$ state of 10, and $G$ is the $G$ conjugation operator.

We can also form the linear combination with positive $G$ parity:

$$U=(1/\sqrt{2})(\psi+G\psi). \tag{18b}$$

There is a particle mixing between $U$ and $V$ analogous to the $K_1^0$, $K_2^0$ situation in the weak interactions. Specifically $U$ and $V$ have different decay modes and slightly different masses. According to SU(3) symmetry the other members of the tenfold system should exist if $U$ snd $V$ are found. These correspond to the states

$$I=0,\quad I=\tfrac{1}{2},\quad I=\tfrac{3}{2},$$
$$S=\pm 2,\quad S=\pm 1,\quad S=\pm 1.$$

It is tempting to associate $V$ with the recently observed[9] peaking in $\pi\rho$ scattering at about 1250 MeV. Similarly, $U$ may be associated with the $B$ and possibly also the[10] $f_0$ resonance at about 1220 MeV. This identification agrees with the $1^-$ assignment of $B$ by some authors.[11]

The consequences of this situation are discussed in more detail separately.[8]

## ACKNOWLEDGMENTS

One of the authors (J. S.) is pleased to acknowledge stimulating conversations with S. Pepper, Y. Ueda, and S. Wong.

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